Hi!

D. Gómez-Castro

ICMAT Postdoc Day,
3rd December 2024

Universidad Autónoma de Madrid
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Short vita

2015–2017 PhD @ U. Complutense. J.I. Díaz

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2024–	“Ramón y Cajal” postdoc @ U. Autónoma 2024

Aggregation-Diffusion equations:

$\partial_t \rho = div \left(\mathrm{m}(\rho) \nabla (U'(\rho) + V + W * \rho) \right)$

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J. A. Carrillo, D. Gómez-Castro, and A. Fernández-Jiménez. **"Partial Mass Concentration for Fast-Diffusions with Non-Local Aggregation Terms".** *Journal of Differential Equations* 409 [2024], pp. 700–773

J. A. Carrillo, D. Gómez-Castro, Y. Yao, and C. Zeng. **"Asymptotic simplification of Aggregation-Diffusion equations towards the heat kernel".** *Archive for Rational Mechanics and Analysis* 247.11 [2023]

Survey:

D. Gómez-Castro. **"Beginner's guide to Aggregation-Diffusion Equations".** *SeMA Journal* [2024]. arXiv: `2309.13713 [math.AP]`

Non-local diffusion: $(-\Delta)^s$ and ∂_t^α

$$(-\Delta)^s u(x) = \int_{\mathbb{R}^d} \frac{u(x) - u(y)}{|x - y|^{d+2s}} dy.$$

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H. Chan, D. Gómez-Castro, and J. L. Vázquez. **“Singular solutions for space-time fractional equations in a bounded domain”.** *Nonlinear Differential Equations and Applications* [2024]

N. Abatangelo, D. Gómez-Castro, and J. L. Vázquez. **“Singular boundary behaviour and large solutions for fractional elliptic equations”.** *Journal of the London Mathematical Society* 107 [2 2023], pp. 568–615

J. I. Díaz, D. Gómez-Castro, and J. L. Vázquez. **“The fractional Schrödinger equation with general nonnegative potentials. The weighted space approach”.** *Nonlinear Anal.* 177 [2018], pp. 325–360

Fractional Sobolev spaces: $W^{s,p}$ and $\dot{W}^{s,p}$

$$[u]^p_{W^{s,p}(\Omega)} = C(d,s,p) \int_\Omega \int_\Omega \frac{|u(x) - u(y)|^p}{|x-y|^{d+sp}} dxdy$$

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L. Brasco, D. Gómez-Castro, and J. L. Vázquez. **"Characterisation of homogeneous fractional Sobolev spaces".** *Calculus of Variations and Partial Differential Equations* 60.2 [2021]

F. del Teso, D. Gómez-Castro, and J. L. Vázquez. **"Estimates on translations and Taylor expansions in fractional Sobolev spaces".** *Nonlinear Anal.* 200 [2020]

H. Brezis and D. Gómez-Castro. **"Rigidity of optimal bases for signal spaces".** *Comptes Rendus Mathematique* 355.7 [2017], pp. 780–785

A new adventure: stochastic ODEs in Finance

Let S_t be the price of an asset

$$dS_t = S_t\left(r\,dt + \sqrt{V_t}dW_t\right)$$

and W_t a Brownian motion w.r.t. the risk-free measure.

- Black-Scholes (1973): $V_t = \sigma^2$ constant.

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- Heston (1993): consider B_t a second Brownian motion with $\langle dW_t, dB_t \rangle = \rho$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dB_t,$$

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- Rough Heston (2014):

$$V_t = V_0 + \int_0^t \kappa(\theta - V_s)K_\alpha(t-s)ds + \int_0^t \sigma\sqrt{V_t}K_\alpha(t-s)dB_s.$$

Several problem related to RH involve fractional ODEs.

Some history

Homegenization

$$\begin{cases} -\,div(A_\varepsilon \nabla u_\varepsilon) = f_\varepsilon & \text{in } \Omega \setminus G_\varepsilon \\ \partial_\nu u_\varepsilon = g_\varepsilon & \text{on } \partial G_\varepsilon \end{cases} \qquad \text{where } G_\varepsilon = \bigcup_{j \in \mathbb{Z}^d} (\varepsilon j + a_\varepsilon G_0)$$

J. I. Díaz, D. Gómez-Castro, and T. A. Shaposhnikova. **Nonlinear Reaction-Diffusion Processes for Nanocomposites.** Berlin, Boston: De Gruyter, 2021

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Rearrangement

F. Brock, J. I. Díaz, A. Ferone, D. Gómez-Castro, and A. Mercaldo. **"Steiner symmetrization for anisotropic quasilinear equations via partial discretization".** *Annales de l'Institut Henri Poincaré C, Analyse non linéaire* 38.2 [2021], pp. 347–368

Questions, comments, remarks?

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